IDA PAPER P-2267

EQUIVALENT UNITS

Stephen J. Balut, *Project Leader*

Thomas R. Gulledge

Norman Keith Womer

August 1989

IDA

INSTITUTE FOR DEFENSE ANALYSES

1801 N. Beauregard Street, Alexandria, Virginia 22311-1772

IDA Log No. HQ 89-34659

DEFINITIONS

IDA publishes the following documents to report the results of its work.

Reports

Reports are the most authoritative and most carefully considered products IDA publishes. They normally embody results of major projects which (a) have a direct bearing on decisions affecting major programs, or (b) address issues of significant concern to the Executive Branch, the Congress and/or the public, or (c) address issues that have significant economic implications. IDA Reports are reviewed by outside panels of experts to ensure their high quality and relevance to the problems studied, and they are released by the President of IDA.

Group Reports

Group Reports record the findings and results of IDA established working groups and panels composed of senior individuals addressing major issues which otherwise would be the subject of an IDA Report. IDA Group Reports are reviewed by the senior individuals responsible for the project and others as selected by IDA to ensure their high quality and relevance to the problems studied, and are released by the President of IDA.

Papers

Papers, also authoritative and carefully considered products of IDA, address studies that are narrower in scope than those covered in Reports. IDA Papers are reviewed to ensure that they meet the high standards expected of refereed papers in professional journals or formal Agency reports.

Documents

IDA Documents are used for the convenience of the sponsors or the analysts (a) to record substantive work done in quick reaction studies, (b) to record the proceedings of conferences and meetings, (c) to make available preliminary and tentative results of analyses, (d) to record data developed in the course of an investigation, or (e) to forward information that is essentially unanalyzed and unevaluated. The review of IDA Documents is suited to their content and intended use.

The work reported in this publication was conducted under IDA's Independent Research Program. Its publication does not imply endorsement by the Department of Defense or any other Government Agency, nor should the contents be construed as reflecting the official position of any Government Agency.

This Paper has been reviewed by IDA to assure that it meets the high standards of thoroughness, objectivity, and appropriate analytical methodology and that the results, conclusions and recommendations are properly supported by the material presented.

UNCLASSIFIED

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
August 1989

3. REPORT TYPE AND DATES COVERED
Final Report, Oct 88 – Aug 89

4. TITLE AND SUBTITLE
Equivalent Units

5. FUNDING NUMBERS
Independent Research

6. AUTHOR(S)
Stephen J. Balut, Thomas R. Gulledge, and Norman Keith Womer

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Institute for Defense Analyses
1801 N. Beauregard Street
Alexandria, VA 22311-1772

8. PERFORMING ORGANIZATION REPORT NUMBER
IDA-P-2267

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
N/A

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12A. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12B. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

This paper presents three ways to estimate cost progress on the same program. One is the conventional cost progress curve using lot data. The other two use periodic expenditure data to estimate the same underlying cost progress curve. The latter methods associate "equivalent units" of output with periodic expenditures. These alternatives also offer opportunities to estimate the underlying curve much sooner than otherwise. Applications of the theory to cost monitoring are discussed.

14. SUBJECT TERMS
Cost Progress Curves, Costs, Weapon Systems, Learning Curves

15. NUMBER OF PAGES
19

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT
Unclassified

18. SECURITY CLASSIFICATION OF THIS PAGE
Unclassifed

19. SECURITY CLASSIFICATION OF ABSTRACT
Unclassifed

20. LIMITATION OF ABSTRACT
SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

UNCLASSIFIED

IDA PAPER P-2267

EQUIVALENT UNITS

Stephen J. Balut, *Project Leader*

Thomas R. Gulledge
Norman Keith Womer

August 1989

INSTITUTE FOR DEFENSE ANALYSES

IDA Independent Research Program

PREFACE

This research was conducted by the Cost Analysis and Research Division (CARD) of the Institute for Defense Analyses (IDA). Funding was provided by the IDA Central Research program. Thomas R. Gulledge and Norman Keith Womer contributed to the research while acting in the capacity of consultants to IDA.

This paper has been reviewed by Stanley A. Horowitz, Bruce R. Harmon, Joseph A. Arena, and Daniel B. Levine of IDA.

CONTENTS

FIGURES

TABLES

I. INTRODUCTION

This section presents background information on the use of the cost progress curve and how it is normally estimated; then, alternative views of the same curve are described that have application in cost monitoring.

A. BACKGROUND

The DoD budgets for purchase of weapon systems on an annual basis and contracts for such systems in lots. The lot size is the number of systems procured (systems authorized, funds appropriated, and obligated) with a given fiscal year's budget. Procurement of a weapon system normally involves budgeting for a sequence of annual lots with the sum of the units in each representing the total quantity required.

Large weapon systems take several years to manufacture. For example, aircraft usually take between 2 and 4 years and ships up to 7 years. Delivery of completed units from each lot begins several years following contract start for that lot. When a sequence of annual lots are contracted for, several lots will be "in process" simultaneously. For example, final assembly of units in lot 1 may be occurring at the same time components for lot 2 are being fabricated.

B. THE COST PROGRESS CURVE

The costs of manufacturing weapon systems are characterized by learning. That is, it has been demonstrated empirically [1] that unit labor hours tend to decline in a systematic way that can be approximated using a simple power function:

$$c(q) = aq^b, \tag{1}$$

where c(q) is the number of hours required to produce unit q, a is the labor hours required to produce the first unit, and b is a parameter that measures the amount of learning reflected in the data used to estimate the model parameters.

The learning curve method is simple, requires very little data, and is easy to apply and explain. These features have led to its widespread acceptance and use. When the method is applied to represent and/or predict the cost/price of subsequent units, the fitted power function is referred to as a cost/price improvement curve.

For systems such as aircraft, cost data are not generally collected on a unit-by-unit basis, rather data are usually available by lot. Using lot data, the parameters, a and b, in equation (1) are estimated. The accepted procedure calls for calculating, for each lot, an ordered pair, the first element of which is the appropriate plot point for the lot on the "q" or quantity axis. (This plot point is associated with the lot midpoint. The procedure for calculating lot midpoints will be discussed later). The second element of the ordered pair is the average unit cost for the lot (the total cost of the lot divided by the number of units in the lot). These data (i.e., the ordered pairs, one for each lot) are used to determine least squares estimates for the parameters a and b.

C. ANOTHER VIEW OF THE COST PROGRESS CURVE

In this paper we show that cost improvement on large weapon systems can be modeled using period (e.g., annual) data and that the representation is equivalent to that provided by lot data. That is, the same curve is representative of output by both lot and time period.

It will be shown that exactly the same curve can be associated with two alternative sets of ordered pairs where the abscissa describes the appropriate lot midpoints and the ordinate is associated with:

1. expenditures by lot by time period (e.g., fiscal year), or
2. total expenditures across all lots within a given time period (e.g., fiscal year).

In the usual method of calculating the ordered pairs, the abscissa gives an unambiguous measure of output. It is associated with a particular number of units - the number of units in the lot. Time periods are not involved in the calculations. In the latter two methods, the relationship between the abscissa and units is not as direct. Time periods are involved. Consider, for example, a program in which manufacturing takes three years. Expenditures (and effort) occur in each of the three years, yet no completed units are delivered in year 1, and perhaps not in year 2. This does not imply that there has not been any output during the first two years. If output is related to effort and apportioned to time periods, it is not appropriate to say all output occurred during year three (and none in years 1 and 2). The two alternative methods for calculating the abscissa presented in this paper provide measures of output in terms of "equivalent units" produced during specified time periods, even though no whole units are produced within the time periods.

D. MONITORING COST PROGRESS

Following a discussion of application to cost monitoring, Section II presents the methods for computing equivalent units that are associated with specified time periods. Section III provides two examples that illustrate the different representations of the same hypothesized cost experience.

The theory of Section II offers an opportunity to use reports of early, actual cost experience to estimate a program cost progress curve years before it would otherwise be possible using current procedures. This has direct application to the task of timely review and monitoring of cost experience on weapon system acquisition programs.

Current procedures for monitoring cost experience on individual contracts (i.e., lots) involve comparisons of budgeted versus actual times and costs. These procedures do not systematically describe cost experience across a sequence of contracts for the same item (e.g., a sequence of annual contracts for a tactical aircraft, or an aircraft engine).

Cost analysts charged with monitoring progress on a program consisting of a sequence of lots (annual contract quantities) must wait until several lots have been completed before the parameters of the underlying cost progress curve for the program can be estimated from lot data. Costs are subsequently monitored by comparing the estimated curve to the Program Manager's forecasts. As additional lots are completed, the parameters of the underlying curve are reestimated, and the comparisons are repeated.

Using the procedure presented in the next section, the underlying cost progress curve for the entire program, consisting of a sequence of lots, can be estimated using period (e.g., annual) data which are routinely reported by contractors. The view taken with periodic data provides information about the underlying cost progress curve sooner, more often, and on several lots simultaneously.

II. THE EQUIVALENT UNITS CONCEPT

This section presents three sets of relationships that are central to understanding the concept of equivalent units. The first subsection describes basic relationships that are employed when applying the unit learning curve to lot data. The last two subsections describe our methods for calculating equivalent units. The first of these methods calculates equivalent units by time periods within lots and the second by time period across all lots in process.

The following notation is used throughout the section:

q = the sequence or unit number of the product,

$c(q)$ = the cost of unit q,

Q_i = the cumulative quantity produced through the end of the ith production lot,

C_i = the cost of lot i,

M_i = the lot midpoint for lot i,

W_j = the fraction of C_i that is spent in year k where $j = k-i+1$,

$E_{ik} = W_{k-i+1}\, C_i$, the expenditure on lot i in year k,

$E_k = \sum_i E_{ik}$, the expenditures in period k,

x_{ik} = the equivalent number of units produced in lot i in year k.

$x_k = \sum_i x_{ik}$, the equivalent units associated with period k,

y_{ik} = the cumulative number of equivalent units attributed to lots prior to lot i and years through k on lot i,

N_{ik} = the plot point associated with E_{ik},

a = first unit cost, and

b = a learning curve slope parameter.

A. THE UNIT LEARNING CURVE AND LOT DATA

The basic relationship between cost and quantity is the unit learning curve [equation (1)]. However, c(q) and q are not observable. In our environment, we observe C_i and Q_i; hence our modeling relationship is

$$C_i = \sum_{q=Q_{i-1}+1}^{Q_i} aq^b. \tag{2}$$

For this theoretical presentation, we assume that a and b are known; later we discuss the implications of estimating a and b from historical data.

To graphically display the information contained in equation (2), average lot costs are plotted against the lot midpoints. We derive the lot midpoints using the approximation suggested by Camm et al. [2]. That is, total lot costs are

$$C_i = \int_{Q_{i-1}+.5}^{Q_i+.5} a\,q^b \tag{3}$$

$$= a\,q^{b+1}/(b+1)\Big|_{Q_{i-1}+.5}^{Q_i+.5}$$

$$= [a/(b+1)]\left[(Q_i + .5)^{b+1} - (Q_{i-1} + .5)^{b+1}\right].$$

Average lot costs are computed by dividing by $Q_i - Q_{i-1}$, and the midpoint for lot i is calculated by solving

$$[a/(b+1)]\left[(Q_i + .5)^{b+1} - (Q_{i-1} + .5)^{b+1}\right] / \ (Q_i - Q_{i-1}) = aM_i^b$$

for M_i as

$$M_i = \left\{\left[\left(Q_i + .5\right)^{b+1} - \left(Q_{i-1} + .5\right)^{b+1}\right] \Big/ \left[\left(Q_i - Q_{i-1}\right)(b+1)\right]\right\}^{1/b}. \tag{4}$$

The ordered pairs $[C_i/(Q_i - Q_{i-1}), M_i]$ are usually plotted in log space as shown in the example in Figure 1.

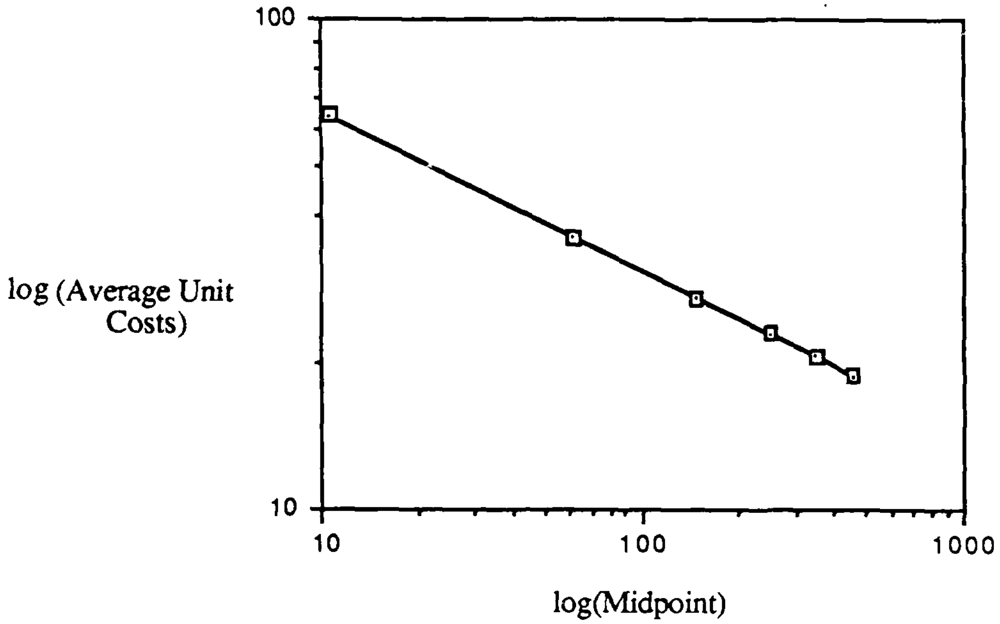

Figure 1. Average Unit Costs Versus Lot Midpoint

The relation

$$C_i/(Q_i - Q_{i-1}) = a\, M_i^b \tag{5}$$

is often used to estimate the learning curve parameters from lot data. Liao [4] describes this approach in detail (also see Gallant [3] and Womer and Patterson [5] for alternative approaches to estimation from lot data).

B. EQUIVALENT UNITS BY TIME PERIODS WITHIN LOTS

Expenditures on a single lot that occur across a sequence of time periods can be related to quantity in such a way that the total number of units in the lot are apportioned to the time periods over which the units were produced (i.e., time periods are assigned "equivalent units"). Further, the method of apportioning reflects the learning (i.e., cost progress) represented in the lot data.

If equations (1) through (5) describe a production program, yearly expenditures for each lot are $E_{ik}=W_{k-i+1}\, C_i$, and we assume the sum of W_j over $_j$ equals one; i.e., all lot costs are expended.

We equate the area under the relevant portion of the learning curve to the yearly expenditure on a lot. The distance on the units axis that is necessary to produce equality is our measure of equivalent units. For each lot i, equivalent units are apportioned by solving the following expression for x_{ik} for each period k for which E_{ik} is positive:

$$E_{ik} = a\int_{y_{i,k-1}}^{y_{i,k-1}+x_{ik}} q^b dq. \tag{6}$$

Note that $y_{ik} = y_{i,k-1} + x_{ik}$.

Given the equivalent units, the plot points for yearly expenditure are calculated as in equation (4) as

$$N_{ik} = \left\{\left[\left(y_{i,k} + .5\right)^{b+1} - \left(y_{i,k-1} + .5\right)^{b+1}\right] / (b+1)\, x_{ik}\right\}^{1/b} \tag{7}$$

for $i \neq k$, and

$$N_{ik} = \left\{\left[\left(y_{i,k} + .5\right)^{b+1} - \left(y_{i-1,k-1} + .5\right)^{b+1}\right] / (b+1)\, x_{ik}\right\}^{1/b} \tag{8}$$

for $i=k$, and $y_{1,0} = y_{0,1} = 0$.

The computation for x_{ik} is precise as long as a and b are known with certainty; i.e., all cost points fall exactly on the curve. If a and b are estimated from historical data, the sum of x_{ik} across all periods in which lot i is in process may not exactly equal the number of units in lot i. For this reason, it is desirable to normalize the apportionments so that the sum of equivalent units associated with a lot exactly equals the precise number of units in the lot. To do this, the following factors are calculated for each lot:

$$F_{ik} = x_{ik} / \left(\sum_k x_{ik}\right).$$

In this case, the equivalent units associated with lot i in period k are redefined as

$$x_{ik} = F_{ik}(Q_i - Q_{i-1}).$$

C. EQUIVALENT UNITS BY TIME PERIODS ACROSS LOTS

Another representation of equivalent units is obtained if units are apportioned to expenditures within time periods, across all lots in process. Let $x_0 = 0$, and consider the following sequence of integral equations, one for each k:

$$E_k = a\int_{x_{k-1}+.5}^{x_k+.5} q^b dq. \tag{9}$$

Midpoints are calculated using equation (4) with Q_k interpreted as cumulative equivalent units through period k. If a and b are known with certainty [that is, if cost experience conforms exactly with equation (1)], this computation directly yields equivalent units. If a and b are estimated from data, a normalization is required to ensure that the sum of equivalent units for the whole program is equal to the number of units actually produced. To do this, the following factors are calculated:

$$G_k = x_k / (\sum_k x_k),$$

and the equivalent units associated with each time period are $x_k = G_k Q$, where Q is the total number of units across all lots.

III. EXAMPLES

The concepts of the preceding section are illustrated for an example program for which data are listed in Table 1. The lot costs were calculated for the given lot quantities using a first unit cost, a, of $135 and an 80 percent slope (b = -.322). The log-log plot of the average lot costs versus the lot midpoints is displayed in Figure 1. Calculating the data in this way avoided the need for a normalization step (as described in Section II.B. and II.C.).

Table 1. Base Program

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Total
Lot quantity	30	70	100	100	100	100	500
Lot Cost	1896	2514	2708	2286	2049	1888	13343
Lot Avg. Cost	63.2	35.9	27.1	22.9	20.5	18.9	
Lot Midpoint	10.6	61.1	146.7	248.3	348.9	449.3	

A. EXAMPLE OF EQUIVALENT UNITS BY TIME PERIODS WITHIN LOTS

For our example program and a (.3, .5, .2) assumed expenditure profile (i.e., values of W_j), the first year's expenditure on lot 1, E_{11}, is 0.3 * 1896 = $568.8. The yearly expenditures for each of the lots, calculated similarly, are displayed in Table 2.

Table 2. Annual Expenditures by Lot

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8
Lot 1	568.8	948.0	379.2					
Lot 2		754.3	1,257.2	502.9				
Lot 3			812.5	1,354.2	541.7			
Lot 4				685.9	1,143.2	457.3		
Lot 5					614.8	1,024.6	409.8	
Lot 6						566.7	944.5	377.8

The number of equivalent units corresponding to E_{11} is calculated from

$$568.8 = \int_{0.5}^{y_{11}+0.5} 135q^{-.322}dq$$

$$= 135/(-.322+1)\left[(y_{11} + .5)^{-.322+1} - .5^{-.322+1}\right]$$

or $y_{11} = 5.8$. Notice that while .3 of the funds planned for lot 1 are spent in the first year, the cost progress curve results in fewer than 0.3 of the lot's units being attributed to the first year. The equivalent units for the second year's expenditure on lot 1 are calculated in a similar manner by solving

$$E_{12} = \int_{y_{11}+0.5}^{y_{12}+0.5} 135q^{-.322}\, dq$$

for y_{12} and defining the equivalent units as $x_{12} = y_{12}-y_{11} = 16.1$. The procedure is repeated to calculate $X_{13} = 8.1$. At this point we note that $y_{13} = x_{11} + x_{12} + x_{13} = 30$ and thus all units associated with lot 1 have been apportioned to expenditures on lot 1. The same will occur for each lot, that is, all units in a lot will be apportioned to expenditures on that lot.

The equivalent units associated with year 2 and lot 2 (the first year on which funds are expended on lot 2) are found by solving

$$E_{22} = \int_{y_{13}+0.5}^{y_{22}+0.5} 135q^{-.322}\, dq$$

for y_{22} and calculating x_{22} as $x_{22} = y_{22}-y_{13}$. These calculations are repeated for each lot and year. The equivalent units by lot and by year for the example program are presented in Table 3, and the average equivalent unit costs are presented in Table 4.

Table 3. Equivalent Units by Lot and by Year

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8
Lot 1	5.8	16.1	8.1					
Lot 2		18.2	35.8	16.0				
Lot 3			27.7	50.6	21.7			
Lot 4				28.6	50.4	21.0		
Lot 5					29.0	50.3	20.7	
Lot 6						29.2	50.2	20.6
Total	5.8	34.3	71.6	94.2	101.1	100.5	70.9	20.6

Table 4. Average Cost of Equivalent Units by Lot and by Year

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8
Lot 1	98.1	58.7	47.1					
Lot 2		41.5	35.1	31.4				
Lot 3			29.4	26.8	24.9			
Lot 4				24.0	22.7	21.8		
Lot 5					21.2	20.4	19.8	
Lot 6						19.4	18.8	18.4

Equations (7) and (8) are used to compute the plot points for the cost values in Table 4. The results are presented in Table 5.

Table 5. Plot Points for Equivalent Units by Lot and by Year

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8
Lot 1	2.7	13.3	26.3					
Lot 2		39.1	65.5	92.3				
Lot 3			114.0	152.5	189.5			
Lot 4				214.6	253.8	289.9		
Lot 5					314.9	354.3	390.1	
Lot 6						415.0	454.5	490.2

Calculating equivalent units in this way yields pairs of plot points that fall exactly on the same cost progress curve as that derived for the base program. Thus the analyst can fit a unit cost progress curve to the data of either Table 1 or Tables 4 and 5 and get exactly the same fitted relation. The fact is illustrated in Figure 2, which displays plots of both the lot data from Table 1 and the period data from Tables 4 and 5.

Note in the example that

$$y_{i,\,i+2} = Q_i . \tag{10}$$

That is, the cumulative equivalent units at the end of each lot are equal to the cumulative units for that lot. This is because the method simply partitions each lot quantity and assigns elements of the partition to expenditures within the lot.

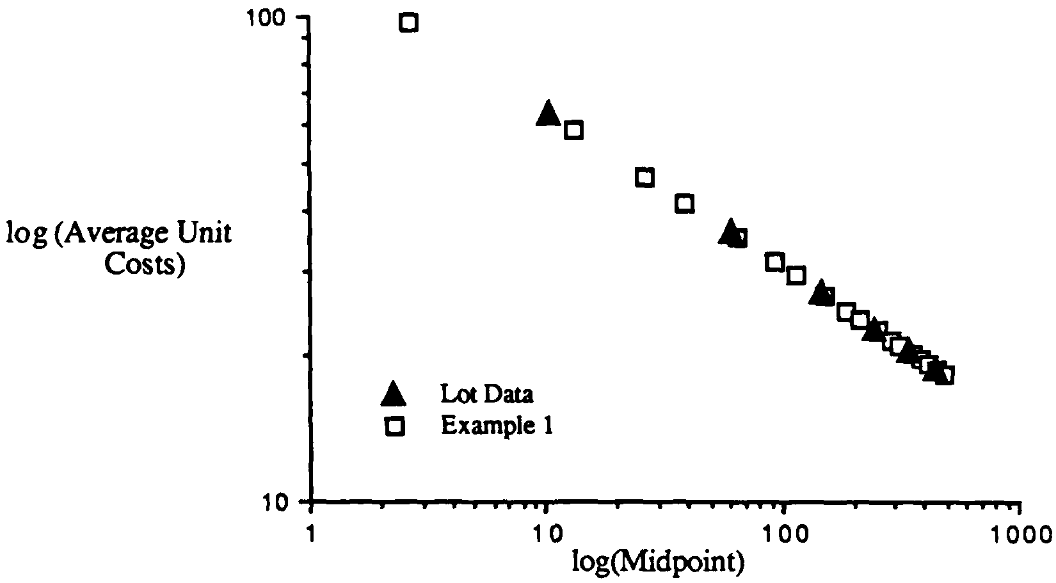

Figure 2. Lot and Period Data for Example 1

A feature of this method for calculating equivalent units has intuitive appeal. When the same lot quantity is purchased over a sequence of years, the apportionment methodology approaches a steady state. That is, the partition of equivalent units across years within a lot approaches the distribution described by weights W_j. This can be seen in Table 3. Lots 3 through 6 have 100 units each. As the lot number increases beyond 3, the partition of each lot's 100 units, as described by the equivalent units apportioned to each of the three years during which the lot is in process, approaches (30, 50, 20). It can be shown that this apparent stabilization as lot number increases is associated with the flattening of the cost progress curve as quantity increases. It can also be shown that the rate of stabilization is related to the slope of the cost progress curve. Note also in Table 3 that as equivalent units stabilize within lots (i.e., across rows), the same occurs within years (i.e., down columns). This results in equivalent units within years (sums down columns) approaching 100. This aligns with intuition, which says both actual and equivalent units per year should equal annual lot size in steady state.

B. EXAMPLE OF EQUIVALENT UNITS BY TIME PERIODS ACROSS LOTS

The annual expenditures by year are the sums of columns in Table 2. The number of equivalent units for year 1 is found by solving

$$E_1 = 568.8 = \int_{0.5}^{x_1 + 0.5} 135q^{-.322}\, dq$$

for $x_1 = 5.8$. The equivalent units for year 2 are found by solving

$$E_2 = 1702.4 = \int_{5.8 + 0.5}^{x_2 + 0.5} 135q^{-.322}\, dq$$

for $x_2 = 32.9$. The results of similar computations for subsequent years are presented in Table 6.

Table 6. Example Summary of Equivalent Units Across Years

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8
Annual Expenditures	568.8	1,702.4	2,448.9	2,543.0	2,299.7	2,048.6	1,354.3	377.8
Annual Equivalent Units	5.8	32.9	71.6	95.6	101.4	100.7	71.4	20.6
Average Annual Cost of Equivalent Units	98.1	51.7	34.2	26.6	22.7	20.3	19.0	18.4
Annual Plot Points	2.7	19.7	71.0	155.4	254.9	356.6	443.6	490.2

The annual plot points in Table 6 were computed using equation (4) and cumulative equivalent units computed from the annual equivalent units shown in Table 6. As in the previous example, plots of ordered pairs consisting of average annual costs of equivalent units and annual plot points lie on the same cost progress curve that was presented in Figure 1. This is demonstrated in Figure 3, where the 6 data points from Figure 1, the 18 data points from Figure 2, and the 8 data points from Table 6 all fall on the same curve.

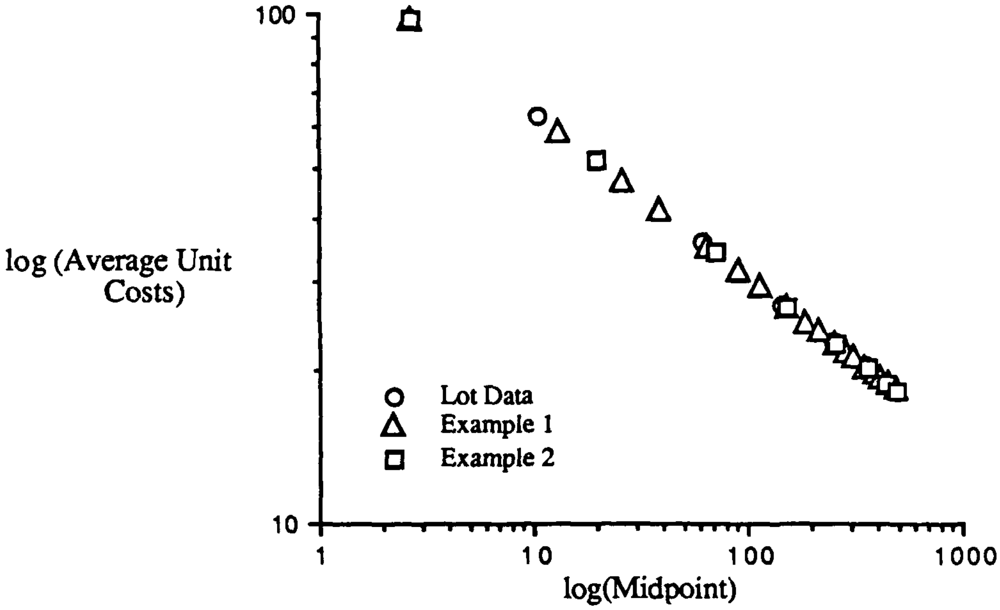

Figure 3. Lot and Period Data for Examples 1 and 2

REFERENCES

[1] Asher, Harold. *"Cost Quantity Relationships in the Airframe Industry"*. The RAND Corporation, R-291, 1956.

[2] Camm, Jeffrey D., James R. Evans, and Norman Keith Womer. "The Unit Learning Curve Approximation of Total Cost." *Computers and Industrial Engineering*, Vol. 12 (1987), pp. 205-213.

[3] Gallant, A. R. "A Note on the Measurement of Cost/Quantity Relationships in the Aircraft Industry." *Journal of the American Statistical Association*, Vol. 63 (1968), pp. 1247-1252.

[4] Liao, Shu S. "Modifications and Extensions: Applying the Learning Curve Formula." *Program Manager*, Vol. 18, No. 3 (1989), pp. 40-46.

[5] Womer, Norman Keith, and J. W. Patterson. "Estimation and Testing of Learning Curves." *Journal of Business and Economic Statistics*, Vol. 1 (1983), pp. 265-272.